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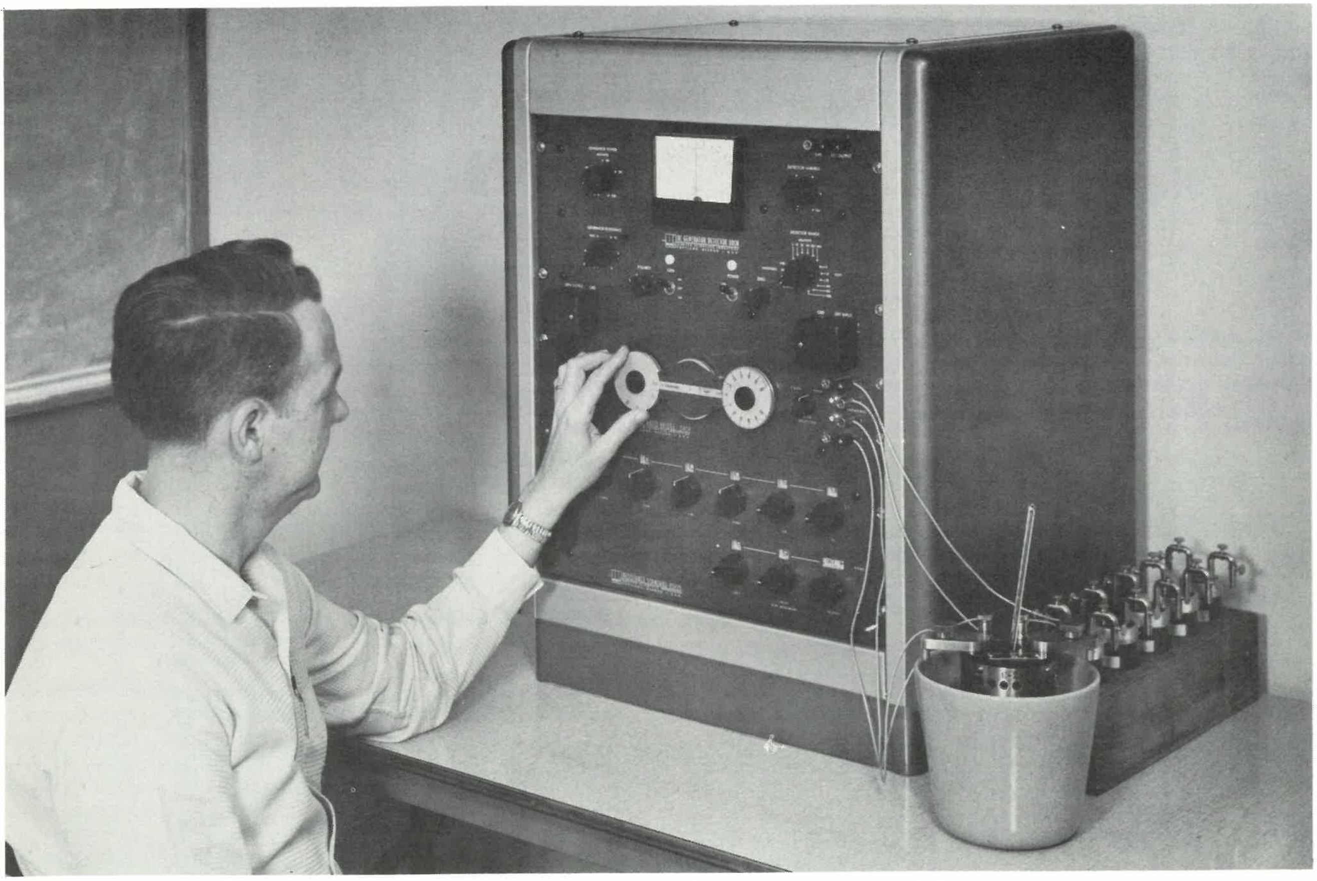

PRECISION RESISTANCE MEASUREMENT—PART I

As the electronics industry becomes more demanding in its requirements for high accuracy components, there is a growing need for the use of more precise resistance measuring equipment and techniques. The accuracy of the standards laboratory must be brought to the production line. Precision measuring equipment must be human engineered to reduce man hour costs and the probability of human error.

We are going to discuss the methods and the ESI equipment for making high accuracy resistance measurements with a minimum of effort and calculation.

Four-terminal techniques are required for measuring low resistances to avoid stray lead and connection resistance errors. Fortunately, both Wheatstone and Kelvin bridges can be designed for four-terminal connections. The Kelvin bridge, however, offers the greatest advantage. Lord Kelvin devised his circuit to measure extremely low resistance values with high accuracy. For many years the Kelvin bridge has been used only for resistors of less than 10 ohms—now its advantages for making precise measurements of higher valued resistors are being realized. With the Kelvin bridge and the four-terminal connection, resistances from a fraction of an ohm up can be measured with four to six place accuracy—this with long, flexible lead wires and clip-on connections.

Three-terminal techniques are required for measuring high resistances to avoid leakage resistance errors. Again, both Wheatstone and Kelvin bridges can be designed to make three-terminal measurements.

ESI provides several resistance measuring systems which incorporate the advantages of three- and four-terminal measurements methods.

The ESI Model 231 Resistance Measuring System uses a Wheatstone bridge connected so that the resistor being measured is guarded from bridge leakage problems. A third terminal is provided on the panel so that external leakage effects can also be cancelled.

The ESI Models 241 and 242 Resistance Measuring Systems use a Kelvin Double Bridge. Four terminals are provided for connecting to the resistor being measured. These bridge systems can be used with the Model SR-1010 Resistance Transfer Standard decades to make transfer measurements. This combination permits a chain of measurements from a single reference standard to any resistance level with

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an accuracy of a few ppm (parts per million). This transfer technique will be discussed in Part II of this article.

All of these systems have a decade accuracy better than ±0.003% and a ratio accuracy better than ±0.003% at the time of certification. This overall accuracy of 0.006% includes a generous safety factor so that the guaranteed accuracy of "0.01% for one year" will normally be maintained for many years. In addition, each range of the Model 230-R Wheatstone Bridge has a screwdriver ratio adjustment. Each ratio can be calibrated at any time to agree with certified standards. The one-to-one range of the Model 240-R Kelvin Bridge also has this ratio adjusting feature.

The Problem of Stray Series Resistance

Figure 1 shows that the total resistance between two terminals includes stray lead and connection resistances.

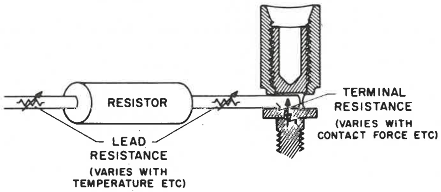
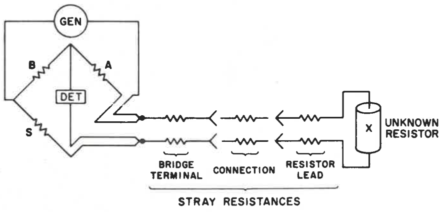

RESISTOR
TERMINAL RESISTANCE (VARIES WITH CONTACT FORCE ETC)
LEAD RESISTANCE (VARIES WITH TEMPERATURE ETC)

Figure 1. Stray resistances.

When making very precise resistance measurements, these strays must be considered. Figure 2 shows these stray resistances in the familiar Wheatstone bridge circuit. If the lead and contact resistances were fixed, we could measure them separately and subtract them from the bridge reading. Unfortunately lead and terminal resistances usually have very high temperature coefficients, so that small changes in room temperature will cause large changes in their values. Also, connection resistance is influenced by dust, oxide, and other contaminants and by the force applied to the terminals. If the resistance to be measured is large relative to the stray resistance, the measurement will be quite accurate, but if the unknown resistance is near the stray resistance in value, large errors in bridge measurement will result.

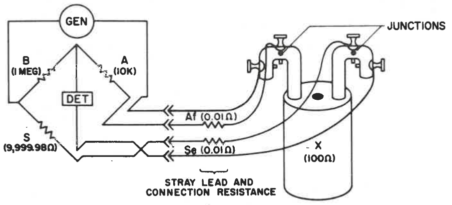


BALANCE EQUATION: $X = S\left(\frac{A}{B}\right) - \text{(STRAY RESISTANCES)}$

Figure 2. Wheatstone Bridge measuring stray resistance.

Four-Terminal Resistance Measurements

In the two-terminal measurement shown in Figure 2, the junction points are at the base of the bridge terminals. In Figure 3 a four-terminal resistance connection is shown. By bringing four leads out from the bridge, the junction points have been moved onto the resistor being measured. This connection places the stray resistances in series with the generator, the detector and the bridge arms A and S. The resistances in series with the generator and detector are not involved in the bridge measurement circuit and will have no effect on the accuracy of the measurement. Resistance in series with bridge arms A and S, however, must be included as a part of resistors A and S. In Figure 3 illustrative values have been assigned to the resistors. With the connection shown, the stray resistances will cause a 2 ppm error. However, with the bridge connected conventionally as in Figure 2, a resistance of 100.02 ohms would be read from the bridge dials. This is a 200 ppm error in the resistance measurement. The lower the resistance being measured, the greater the advantage from the circuit of Figure 3.



BALANCE EQUATION: $X = S(1+e)\left[\frac{A(1+f)}{B}\right]$ $\quad X(1-e-f) \approx S\left(\frac{A}{B}\right)$

Figure 3. Wheatstone Bridge with reduced effect from stray resistance.

Commercial Wheatstone bridges can often be connected for this type of four-terminal measurement. Figure 4 shows these connections for the ESI Model 231 Resistance Bridge and the Model 291 Impedance Bridge.

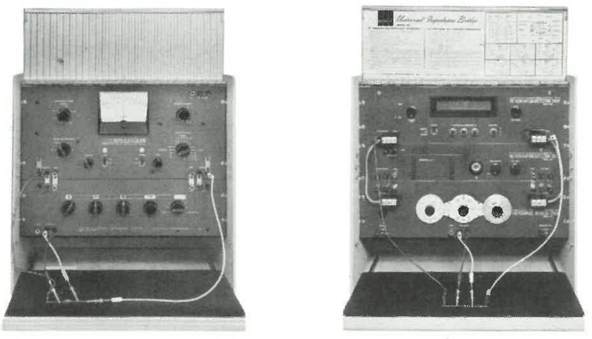


Figure 4. Four-terminal connection using the ESI Model 231 Resistance Bridge and the ESI Model 291 Impedance Bridge.

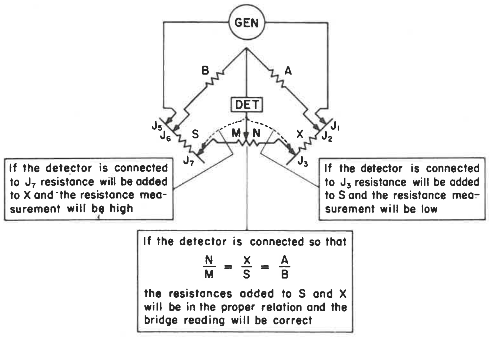
The Kelvin Bridge Circuit

With the Kelvin bridge circuit, four-terminal measurements of even greater accuracy and convenience are possible.

There is one problem with the four-terminal connection to the Wheatstone bridge. One of the stray resistances is in series with a low resistance bridge arm and can cause trouble. With a four terminal connection to the Kelvin circuit all of the stray resistances can be connected in series with high resistance bridge arms or external circuits. Thus all stray resistance problems are minimized.

For maximum sensitivity in low resistance measurements a bridge arm resistor approximately equal to the resistor being measured is required. In the Kelvin circuit this resistor is also connected with four terminals to avoid its lead and contact problems.

Figure 5 shows the first step in the evolution of a Kelvin bridge from a Wheatstone bridge. We want to measure the resistance X between the junctions J_2 and J_3. Both X and S are low resistance values. Let us connect to the tap


GEN
B
A
DET
J5
J6
S
M
N
X
J1
J2
J7
J3
If the detector is connected to J7 resistance will be added to X and the resistance measurement will be high
If the detector is connected to J3 resistance will be added to S and the resistance measurement will be low
If the detector is connected so that N/M = X/S = A/B the resistances added to S and X will be in the proper relation and the bridge reading will be correct

BALANCE EQUATION: $X = S\left(\frac{X}{B}\right) + M\left(\frac{A}{B} - \frac{N}{M}\right)$

Figure 5. Evolution of the Kelvin Bridge from the Wheatstone Bridge.

of a resistor between X and S. If this tapped resistor is kept very small, we have a conventional Wheatstone bridge in which the resistor consists of stray resistances. If the resistor is larger, the position of the detector tap determines the bridge accuracy. By connecting the detector at the J_7 end of this tapped resistor we will get a high reading for our unknown resistor. By connecting it to J_3, we will get a low reading. There is some magic point along the resistor which will give the true resistance reading. To reach this connection point we set N and M in the ratio:

$$\frac{N}{M} = \frac{X}{S} = \frac{A}{B} \tag{1}$$

To show why this is so, let N represent the resistance from J_3 to the detector connection, and M the resistance from J_7 to the detector connection. M and N include all the lead and connection resistance between junction points J_3 and J_7. Now the bridge balance equation is:

$$\frac{A}{B} = \frac{(X+N)}{(S+M)} \tag{2}$$

But we want to get the balance equation:

$$\frac{A}{B} = \frac{X}{S} \tag{3}$$

To do this we need to set the detector tap so that

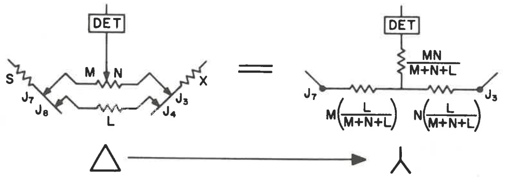
$$\frac{N}{M} = \frac{X}{S}\left(\text{also} = \frac{A}{B}\right) \tag{4}$$

Then we can get the desired balance equation (3) from equation (2)

$$\frac{A}{B} = \frac{X + \frac{MX}{S}}{S+M} = \frac{SX+MX}{S^2+MS} = \frac{X\cancel{(S+M)}}{S\cancel{(S+M)}} = \frac{X}{S} \tag{5}$$

One more step and we will have the Kelvin bridge.

In Figure 6 we have added a resistor, L. L is all of the resistance in its branch between the junctions J_3 and J_7.



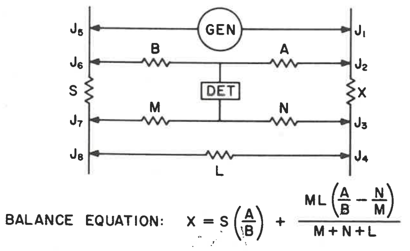
DET
S
M
N
X
J_7
J_8
L
J_4
J_3
=
DET
$\frac{MN}{M+N+L}$
J_7
J_3
$M\left(\frac{L}{M+N+L}\right)$
$N\left(\frac{L}{M+N+L}\right)$
△
⅄

Figure 6. △-Y **transformation.**

We now have a △ circuit between the detector and junction points J_3 and J_7. A △-Y transformation results in the circuit on the right-hand side of Figure 6. We still have a resistor connecting J_3 and J_7 which is tapped in the ratio of N to M. Thus the bridge reading will be correct if:

$$\frac{N}{M} = \frac{A}{B} \tag{6}$$

no matter what the value of L is, from an open circuit to a short circuit. By redrawing this Kelvin bridge circuit in the form of Figure 7, we see that the contact resistances at J_1, J_4, J_5, and J_8 will have no effect on the accuracy of the measurement. Resistances at the other junction points will be in series with the internal bridge arms. Their effects can be minimized by keeping the bridge arm resistances high. Also by making the bridge arm resistances adjustable, these stray resistances can be compensated for. In the Model 240-R, resistors A and N of Figure 7 are made slightly smaller than their nominal values and variable resistors are placed in series with them for lead compensation.



J_5
GEN
J_1
B
A
J_6
J_2
DET
S
X
M
N
J_7
J_3
J_8
J_4
L
BALANCE EQUATION: $X = S\left(\frac{A}{B}\right) + \frac{ML\left(\frac{A}{B} - \frac{N}{M}\right)}{M+N+L}$

Figure 7. Kelvin Bridge.

A look at the balance equation of Figure 7 shows some additional advantages to the circuit. Here, the expression:

$$\frac{ML\left(\frac{A}{B} - \frac{N}{M}\right)}{M+N+L} \tag{7}$$

represents the measurement error. This term will vanish if:

$$\frac{N}{M} = \frac{A}{B} \tag{8}$$

Even if this condition is not met exactly, the term will be very small if L is very small compared to M+N. By using a low resistance bus bar connection from J_4 to J_8 the value of L is kept small. This minimizes the effects of stray resistances at junctions J_3 and J_7. The only stray resistances left are those at junctions J_2 and J_6. By keeping the resistances of A and B high relative to these strays, resistances X and S can be compared with extremely high accuracy.

Kelvin bridges are traditionally used for resistances up to one ohm or, occasionally, to ten ohms. Even above these resistance levels, however, the Kelvin bridge is useful for avoiding trouble from stray series resistance. It should be recognized that even up to the kilohm level you are free from concern over lead and contact resistance even when you use ordinary flexible leads and test clips for all connections.

Kelvin Clips

Spring clips with jaws insulated from each other permit rapid connections for precision four-terminal measurements. These clips have been named "Kelvin Clips" at ESI in honor of Lord Kelvin's contribution to four-terminal measurements. The four-terminal technique prevents any decrease in accuracy even though such clips may have a variable connection resistance of a few milliohms. This is true even when measuring resistances which are lower than the resistance of the clips.

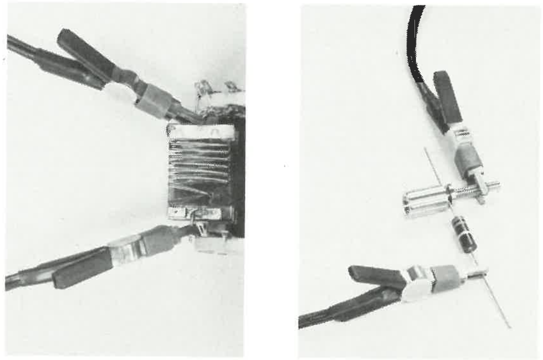


Figure 8. Using Kelvin clips.

The Problem of Stray Shunt Resistance

When a resistor is mounted between two terminals on an insulating block, the leakage resistance through the block shunts the resistor. When separate insulators are mounted on a conducting support, however, as illustrated in Figure 9, the leakages are separated from the resistor. A high value resistor or resistance standard can be mounted in a case using separate insulators as in Figure 9 so that the

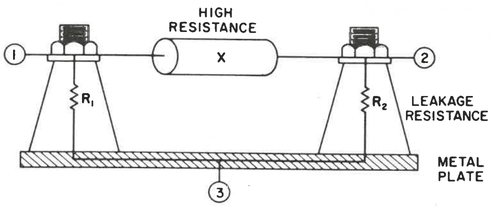


Figure 9. Three-terminal resistor.

insulator leakages can be separated from the resistor value during measurement. When the third terminal (the case or mounting plate) is connected to a Wheatstone bridge as in Figure 10, the leakages are placed across other branches of the bridge circuit. Leakage resistance R_2 is across the detector and will cause no error. Leakage resistance R_1 is across resistor A and will change its value slightly. With good insulation materials the resistance of R_1 can be made so much higher than that of A that this change can be neglected. With this three-terminal connection X can be measured to high accuracy even though its resistance may be many thousands of megohms, and the day very humid. The ESI Model 230-R Bridge measures to 12 gigohms (kilomegohms). The ESI Model 240-R Bridge can be used for even higher values. These bridges are therefore guarded—they are constructed so that insulator leakage paths never appear in parallel with the resistor being measured.

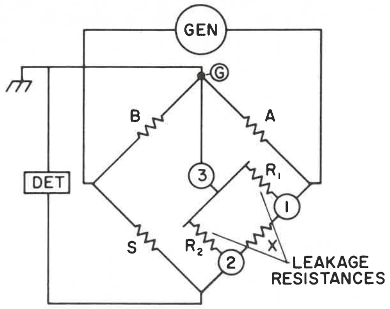


Figure 10. Three-terminal resistor connected to a Wheatstone Bridge.

Figure 11 shows the method of making three-terminal connections when using the Kelvin circuit.

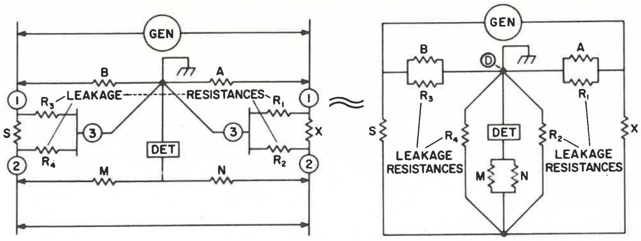


Figure 11. Three-terminal connection for high resistance measurements with a Kelvin Bridge.

Here both the standard S and the unknown X can be connected as three-terminal resistors. When the resistances of X and S are high, the circuit on the right is a good approximation of the measurement condition. Here we see that the leakage resistances R_2 and R_4 are across the detector and will cause no error, while R_1 and R_3 are across relatively low resistance bridge arms. Even here, normal leakage values will not give measurement errors.

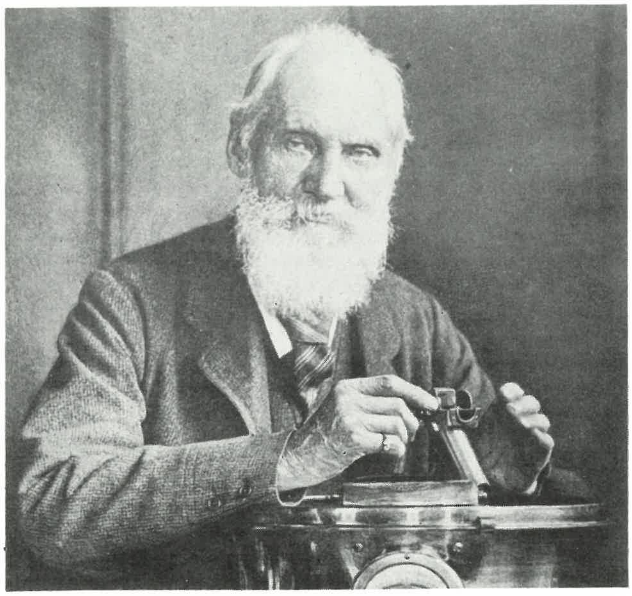
In the ESI Model 240-R Kelvin Ratio Bridge, all of the terminals are mounted on bushings which pass through the metal chassis. By connecting the chassis to point D of Figure 11 all of the bridge leakage is across either the detector or a low resistance bridge arm. Terminals connected to the chassis are provided for connecting the third terminal of three-terminal standard and unknown resistors.

Continued

In the next issue we will continue the discussion of resistance measurement. The subject of resistance transfer from a single reference standard to any resistance level will be described.

Pardon Our Arithmetic

The total shown in Figure 9 of Volume 1, Number 1, should have been 0.972,846, and the total in Figure 10 should have been 0.98.



Baron Kelvin of Largs

Sir William Thompson, Baron Kelvin of Largs (1824-1907), was a truly remarkable scientist in an equally remarkable age. Nineteenth Century science was favored with such men as Faraday, Hertz, Helmholtz, Maxwell, Wheatstone, Edison, Marconi and many other illustrious names. Yet among these Lord Kelvin was generally acclaimed as the leading light. Lord Kelvin himself gave two reasons for this "success"; first, a belief that science had an obligation to contribute much to human welfare, and, second, a strong desire just to "see what the Creator had made." These high aims resulted in a very productive life.

When only 16, he wrote a paper for the Cambridge Mathematical Journal which firmly established Fourier's series. At 21, he showed the connection between Coulomb's Elementary Laws of Electric Charge and Faraday's discovery that electric forces between bodies depend on the nature of the substance between them. This early promise was born out with such later discoveries as the Second Law of Thermodynamics, the Siphon Recorder, the Vortex Theory of Atoms, the Kelvin Marine Compass, and literally hundreds of others. To the British Royal Society he presented "Mathematical Theory of Magnetism" which is still an important reference work today. He laid the mathematical and experimental groundwork for the explanation of oscillation in a circuit containing capacitance, inductance and resistance (the basis of radio).

Probably the best known of Lord Kelvin's contributions was his part in the successful laying of the first transatlantic telegraph cable. For this great undertaking it was necessary to keep the resistance of the copper cable as low as possible. Kelvin found, however, that different companies were producing copper of differing purity which resulted in widely differing values of resistivity. The success of the whole venture depended on obtaining copper of extremely low resistance. In 1860, however, it was hard to measure low resistance. Lord Kelvin overcame the problem in 1861 by inventing the Kelvin Double Bridge.

Litho in U.S.A.